



**GCE AS/A level**

0981/01



S15-0981-01

**MATHEMATICS M2**

**Mechanics**

A.M. FRIDAY, 5 June 2015

1 hour 30 minutes

### **ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

### **INSTRUCTIONS TO CANDIDATES**

Use black ink or black ball-point pen.

Answer **all** questions.

Take  $g$  as  $9.8 \text{ ms}^{-2}$ .

Sufficient working must be shown to demonstrate the **mathematical** method employed.

### **INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The vectors  $\mathbf{x}$  and  $\mathbf{y}$  are given by

$$\begin{aligned}\mathbf{x} &= \sin\theta\mathbf{i} + 2\cos2\theta\mathbf{j}, \\ \mathbf{y} &= 2\mathbf{i} - \mathbf{j}.\end{aligned}$$

Find the values of  $\theta$  between 0 and  $2\pi$  such that  $\mathbf{x}$  is perpendicular to  $\mathbf{y}$ . [6]

2. An object of mass 50 kg moves in a straight horizontal line under the action of a constant horizontal force of magnitude 1600 N acting along the line. The resistance to motion of the object is proportional to time  $t$  seconds. At time  $t$  seconds, the velocity of the object is  $v$  ms<sup>-1</sup> and at time  $t = 2$ , it is moving with velocity 41 ms<sup>-1</sup> and acceleration  $-4$  ms<sup>-2</sup>.

- (a) Show that  $v$  satisfies the differential equation

$$\frac{dv}{dt} = 32 - 18t. \quad [4]$$

- (b) Find an expression for  $v$  in terms of  $t$  and determine the times when the velocity of the object is 28 ms<sup>-1</sup>. [6]

3. A vehicle of mass 6000 kg is moving up a slope inclined at an angle  $\alpha$  to the horizontal, where  $\sin \alpha = \frac{6}{49}$ . The vehicle's engine exerts a constant power of  $P$  W. The constant resistance to motion of the vehicle is  $R$  N. At the instant the vehicle is moving with velocity  $\frac{16}{5}$  ms<sup>-1</sup>, its acceleration is 2 ms<sup>-2</sup>.

The maximum velocity of the vehicle is  $\frac{16}{3}$  ms<sup>-1</sup>.

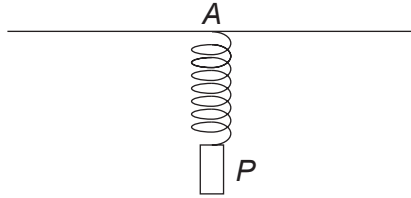
Determine the value of  $P$  and the value of  $R$ . [9]

4. A particle of mass 0.5 kg is moving under the action of a single force  $\mathbf{F}$  N, where  $\mathbf{F} = (4t - 3)\mathbf{i} + (3t^2 - 5t)\mathbf{j}$ .

- (a) The velocity of the particle at time  $t$  s is  $\mathbf{v}$  ms<sup>-1</sup>. When  $t = 0$ ,  $\mathbf{v} = 8\mathbf{i} - 7\mathbf{j}$ . Find an expression for  $\mathbf{v}$  in terms of  $t$ . [5]

- (b) When  $t = 3$ , the particle receives an impulse of  $2\mathbf{i} - 9\mathbf{j}$  Ns. Find the speed of the particle immediately after the impulse. [5]

5. The diagram shows a light spring of natural length  $0.4\text{ m}$  and modulus of elasticity  $1470\text{ N}$  with one end  $A$  fixed and the other end attached to an object  $P$  of mass  $15\text{ kg}$ .



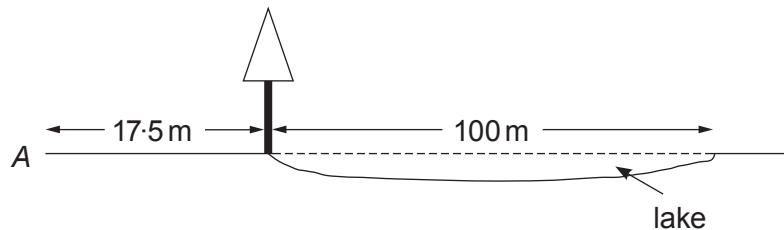
Initially,  $P$  hangs in equilibrium with the spring vertical.

- (a) Determine the extension of the spring. [3]

The object  $P$  is pulled downwards so that the total length of the spring is  $0.56\text{ m}$ . It is then released.

- (b) Calculate the speed of  $P$  when it is at a distance  $0.45\text{ m}$  from  $A$ . [8]

6. A golfer hits a ball from a point  $A$  with initial velocity of  $35\text{ ms}^{-1}$  at an angle  $\alpha$  above the horizontal where  $\sin \alpha = 0.8$ . The ball passes over a tree which is growing in front of a lake. The lake is  $100\text{ m}$  wide, as shown in the diagram. The tree is at a horizontal distance of  $17.5\text{ m}$  from  $A$ .



- (a) Determine whether or not the golf ball will fall into the lake. [6]

- (b) Find the magnitude and direction of the velocity of the ball as it passes over the tree. [7]

7. A car of mass  $1200\text{ kg}$  is moving in a horizontal circle of radius  $80\text{ m}$  on a road banked at an angle of  $12^\circ$  to the horizontal. When the car is moving with a constant speed of  $v\text{ ms}^{-1}$ , there is no tendency to sideslip. Calculate the normal reaction of the road on the car and find the value of  $v$ . [5]

**TURN OVER**

8. One end of a light inextensible string of length  $0.8\text{ m}$  is attached to a fixed point. The other end of the string is attached to a particle  $P$  of mass  $3\text{ kg}$ . Initially  $P$  hangs at rest with the string vertical. The particle  $P$  is then projected horizontally with speed  $5\text{ ms}^{-1}$ , so that it starts to describe a vertical circle. When the string is inclined at an angle  $\theta$  to the downwards vertical,  $P$  has speed  $v\text{ ms}^{-1}$  and the tension in the string is  $T\text{ N}$ .
- (a) Find, in terms of  $\theta$ ,
- (i) an expression for  $v^2$ ,
  - (ii) an expression for  $T$ . [8]
- (b) Find the greatest possible value of  $\theta$  and briefly describe the subsequent motion of  $P$ . [3]

**END OF PAPER**